### **Executive Summaries**

A brief description of each problem is given followed by the equation-free Executive Summary for the problem

#### **FAULT SLIP IN A MINING CONTEXT**

#### **Division of Mining Technology, CSIR**

#### **Industry Representative**

John Napier, Division of Mining Technology, CSIR, Auckland Park Johannesburg.

#### Moderator

Neville Fowkes, School of Mathematics and Statistics, University of Western Australia, Crawley, Western Australia 6009, Australia.

#### Description

Fault slip can result in tunnel collapse and so is a major concern for miners. Such slip can be brought about by seismic activity remote from the mining site or can be locally generated by the mining activity itself; the latter is of interest in this case. Tunnel construction may cause a reduction in the normal force acting on a nearby fault or an increase in the shearing force acting along the fault and thus result in fault slip. Furthermore the effect of the slip along this fault will cause a redistribution of stress throughout the mining site so that other faults may slip or may be further loaded; subsequent mining activity may trigger such faults. The failure may be static in character in the sense that a quasi-steady description of the stress field can be used to identify unsafe faults, or dynamic in the sense that elastic wave propagation isssues need to be taken into account. Further complications arise in that very little relevant information is available about the geology and stress state of the mining area and it is unlikely that such information will ever be practicably available. The problem is very difficult both because the physics is not well understood and because of the absence of data.

The Study Group was asked to investigate the normal and shear stress near a fault due to mining and the condition for the onset of slip at the fault. It was also asked to develop a flexible and robust numerical differencing scheme that has a well defined error behaviour to analyse fault slip.

#### **Executive summary**

No one really knows how best to tackle problems of this kind and in fact one might well argue that the objective of collecting sufficient data to allow rigourous modelling is futile given the complex nature and large Generally, however, it is variability of materials and site conditions. agreed that models should be seen as a numerical laboratory where the engineer experiments with the main variables and parameters, learning about their mutual relations and their influence on global behaviour; this approach has been adopted by the CSIR. Many, many person years have been invested in trying to understand how to deal with the physics and numerics of this problem, the related earthquake problem and the more remote groundwater flow problems; all these problems are dogged by the same data lack/complex physics limitations. What could be attempted in the workshop with the available participants and computational machinery and expertise was very limited. Participants reported on their experience and knowledge in related modelling and computational areas and an attempt was made to survey recent work in the related seismic area. Hopefully the discussions and review articles on the comparative advantage of the range of computational techniques used including particle techniques and recent advances in slip physics models and hazard assessment will be useful for the CSIR. A summary is provided in the final report.

The normal and tangential components of stress at a fault due to an excavation were calculated using the stress distribution for a point force of magnitude equal to the weight of the material excavated per unit area and direction vertically upwards. The normal and tangential components of stress were used to invetigate fault slip due to an excavation.

## TRANSPORTATION OF A WATER BASED SLURRY IN AN OPEN FURROW, LAUNDER OR STREAM

#### **Lynx Geosystems South Africa**

#### **Industry Representatives**

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#### Moderator

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#### Description

A mineral processing plant produces vast amounts of waste material. This is made of rocks of all sizes, ranging from very fine pebbles with a diameter less than a millimetre to rocks with a diameter of up to twenty centimetres. In the set-up presented at the Study Group, the waste is washed away with high pressure water and flows downhill to a pond three kilometres and approximately three hundred meters lower. As it flows the slurry carves a channel, typically one metre by one metre. Since the mining area, where the material is produced, is slowly moved away from the pond, the average angle of the furrow to the horizontal decreases progressively. As the angle decreases the possibility of large rocks becoming stuck in the channel increases. The Study Group was asked to investigate the effect of changing the channel slope with the aim of ensuring that all waste would be washed away.

#### **Executive summary**

In the current operating conditions, the flow is very turbulent and the water flux through the channel is very high. For these reasons, small particles, that represent most of the material produced, will always be carried downstream to the pool. Should they deposit on the bottom of the channel, they will be immediately re-entrained. Hence a flow model was used in which the small particles were assumed to be well-mixed with the water. This means that the particle/water mixture may be considered as a single

fluid, whose properties such as the density and dynamic viscosity are given in the literature.

Focus was then turned on to the largest rocks. The force applied by the flow of the fluid mixture on a rock may be encapsulated in the Shields number, the ratio of the shear stress applied on the boulder to its weight. This may be expressed as a function of the fluid velocity, density and dynamic viscosity using standard turbulence models. The minimum value of the Shields number required for rocks to take off from the ground may be found in the literature. It varies with the diameter of the rock and its shape through a parameter known as the angle of repose. This is the inclination of a plane at which the rock placed on the plane would remain at rest or if in motion would roll or slide down. This links the geometry of the rocks to the flow velocity, which in turn may be related to the angle of the furrow using Manning's model, for example. The minimum angle of the furrow necessary to transport a given rock all the way down to the pond may then be evaluated. Numerical testing will be included in the final report but the initial simulations provided results that were consistent with experimental observations.

## FRACTURING ROCK WITH ULTRA HIGH PRESSURE WATER

#### **Mining Industry**

#### **Industry Representatives**

Edward Moss and Jian Cheng, School of Mechanical, Industrial and Aeronautical Engineering, University of the Witwatersrand.

#### Moderator

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#### Description

The problem is based on existing methodologies for oil wells and pertains to potential new techniques in gold mining. The investigation is concerned with the fracturing of rock through the growth of cracks by ultra high pressure water and has the advantage that there is no dust, is energy efficient and evacuation of the mining area is not needed. A strong and rapid pressure pulse of magnitude in the range 250 to 1000 MPa and of time duration 0.5 to 2m sec in a water filled borehole, formed by a small quantity of propellant or by mechanical means, causes the formation of rapidly propagating micro cracks which grow longer and thicker with time because of the high pressure liquid within them. As this occurs the volume of the total cavity increases which, combined with strong viscous effects, dissipates the pressure with space and time while separating the portions of the rock further. During the initial stages of the process, the separation distance of the crack compared with its length will be small and therefore lubrication theory could be considered. The compressibility of liquids at sufficiently high pressure may also be significant. The Study Group was asked to develop a mathematical model for the development of the cracks.

#### Esecutive summary

Very little consideration was given to the details of how the distrubance initiated by the propellant travels down the borehole. It was simply assumed that borehole conditions for the crack are "known". As long as the temperature is elevated above ambient, the phase diagram of water shows conclusively that the water will not change phase to steam.

It is easy to show that even at the very large pressures envisaged during the process, the compressibility of the water involved will be negligible. It is thus justifiable to treat the water as an incompressible viscous fluid.

Some fairly simple fracture mechanics theory may be employed to give an idea of the initial crack length that is likely to be formed by a given pressure.

Standard lubrication theory shows that the crack width h(x,t) and the fluid pressure p satisfy a conservation law of the form

$$\frac{\partial h}{\partial t} - \frac{1}{3} \frac{\partial}{\partial x} \left( h^3 \frac{\partial p}{\partial x} \right) = 0.$$
 (1)

The problem must now be closed by adding another equation that relates  $p_x$  to h using the elastic nature of the rock. The natural way to close this problem is to use standard crack theory. Solving the elastic problem for a Griffith crack gives

$$p = K \int_{0}^{L(t)} \frac{h'(s)}{x - s} ds$$

where L(t) is the crack length and K an elastic constant. Unfortunately this is not an easy relationship to work with, though similarity solutions have been derived in the literature.

Careful estimates of time, length, viscosity and velocity time scales appear to indicate that it is likely that the "impulsive lubrication theory" equations (i.e. with an added  $u_t$  term in the x-momentum equation, but no  $uu_x$  term) should be used. In these circumstances the flow problem can still be solved and an equation for h(x,t) equivalent to the nonlinear diffusion equation above, although a great deal more complicated, may be derived.

All of the modelling discussed during the week assumed that the crack was completely filled with water. It is possible, however, that an air cavity exists ahead of the water. It is not currently clear how this could be taken into account.

# MODELLING SURFACE HEAT EXCHANGES FROM A CONCRETE BLOCK INTO THE ENVIRONMENT

#### **Cement and Concrete institute**

#### **Industry Representatives**

Yunus Ballim and Anthony Patini, School of Civil and Environmental Engineering, University of the Witwatersrand.

#### Moderator

Tim Myers, Department of Mathematics and Applied Mathematics, University of Cape Town, Private Bag, Rondebosch 7701, Cape Town.

#### Description

In order to determine the temperature development in a concrete dam during the construction stages, it is necessary to specify appropriate boundary conditions on the surface of the concrete. The current method is to assume that the surface is maintained at the ambient temperature. The ambient temperature is taken from data supplied by the local meteorological office. The advantage of this model is that it is simple and uses data that is readily available. The disadvantage is that the model is over simplified since it ignores a number of important effects such as cloud cover, wind and solar radiation. However, increasing the complexity of the boundary condition also requires more input data which may be difficult to collect. The Study Group was asked to develop a temperature prediction model with more realistic boundary conditions.

#### Executive summary

An investigation was carried out to determine the effect of applying a new, more realistic boundary condition involving radiation, con-vection and evaporation under a variety of weather conditions. The magnitude of the various terms in the new boundary condition was also investigated to determine which are dominant and which may be neglected.

Numerial simulations were carried out to compare the two models. These simulations are continuing and details will be included in the final report, however, results so far obtained indicate that the effect of 'pegging' the surface temperature to the ambient level will overestimate the heat exchanges between the concrete and the air. As one might expect the errors are relatively large near the surface, they decay exponentially with distance from the surface in a length scale of 14cm, although relatively small persistent errors can occur.

Additionally a simple model was developed to take into account the drying out of the surface. The large change in moisture diffusivity, vapour verses liquid transport, brought about by drying is an essential component of the model which determines the change in thickness of the dry zone. Accounting for this zone seems to be important both because of its effect on global heat and moisture transport and because of the reduced hydration in this layer. A simplified effective boundary condition applied at the surface of the concrete which takes into account this drying process is being developed. Simulations will be included in the report.

#### **MATURITY EFFECTS IN CONCRETE DAMS**

#### **Cement and Concrete Institute**

#### **Industry Representatives**

Yunus Ballim and Anthony Patini, School of Civil and Environmental Engineering, University of the Witwatersrand.

#### Moderator

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#### Description

Combining water and cement produces an exothermic reaction in which heat is liberated. The reaction is referred to as hydration and to describe the process of hydration the concept of maturity has been introduced. Dam walls are built sequentially in discrete blocks to allow for shrinkage. The lower layer of cement has not lost all of its heat before the upper layer is added. The upper layer transmits its heat into the lower layer increasing its temperature and hence the rate of cement hydration. The thermal exchanges between the two layers can induce undesirable cracking. A better understanding of the underlying processes may lead to better construction practices. The Study Group was asked to develop a model of the process incorporating maturity, water content and thermal exchanges.

#### Executive summary

A simple model that takes into account the different levels of hydration, or maturity, in the two levels has been suggested by Ballim and Graham and this may well be the best model to account for the above observations. Study has commenced on a more complete model incorporating maturity, water content and thermal exchanges within the concrete in a simplified one-dimensional geometry, and simulations are in progress. The aim of this work is not to produce a more complex model but to identify the simplest model required to explain the observations. Preliminary work on this problem will be reported in the final report.

#### PIPED WATER COOLING IN CONCRETE DAMS

#### **Cement and Concrete Institute**

#### **Industry Representatives**

Yunus Ballim and Anthony Patini, School of Civil and Environmental Engineering, University of the Witwatersrand.

#### **Moderators**

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Tim Myers, Department of Mathematics and Applied Matheamtics, University of Cape Town, Private Bag, Rondebosch 7701, Cape Town.

#### Description

Concrete is a poor conductor of heat and therefore during the construction period only a small amount of hydration heat is lost from the exposed surface. The larger the slab the less the relative loss of heat will be. A pipe network is often included in the concrete during construction and chilled water is piped through the concrete in order to extract hydration heat from the concrete. After the concrete has cooled sufficiently the pipes are filled with concrete. The Study Group was asked to incorporate the pipe network into a concrete temperature model and determine an optimal pipe network.

#### **Executive summary**

By using a simple model of flow of water through a pipe surrounded by hydrating concrete, it was shown that in the steady state the temperature increases linearly along the pipe. Recommendations were made on pipe length, inlet temperature and discharge rate through the pipe.

### DISCRIMINATION AND IDENTIFICATION OF UXO USING AIRBORNE MAGNETIC GRADIENTS

#### **Image Processing Company**

#### **Industry Representative**

Neil Pendock, School of Computational and Applied Mathematics, University of the Witwatersrand.

#### **Moderator**

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#### **Description**

The safe detection and removal of unexploded ordinances (UXO) such as landmines is a serious problem in Africa. Vector measurements of the earth's magnetic field from a helicopter may be used to infer the locations of some types of landmines using remote sensing. The inverse problem is to estimate the locations and magnetic moments of a set of dipoles that model the measured data. The Study Group was asked to imporve on existing algorithms.

#### Executive summary

The problem proposed was to identify possible UXO targets in magnetic gradient data measured from the air. Frahm (1972) describes the inversion of the magnetic gradients to solve for the magnetic moments of the dipoles and the direction of the dipoles. The method solves for a single dipole from each individual set of gradients and results in four possible solutions. The four solutions are symmetrical and so two solutions can be ignored as they appear above the sensor. The approach taken by the Study Group uses multiple sets of gradient points to solve for the dipoles. Some assumptions were made in order to simplify the model, which will be explained later. Then it was tested on a few constructed cases and some

real data. The results of the tests showed that the model can work but it is not necessarily very good for real data.

In order to simplify the model, the directions to the dipoles were fixed by placing them along the ground under the path of the sensor. By having the positions of the dipoles fixed, the problem - solving for the values of the magnetic gradients at each dipole - using multiple data points and dipoles is a system of linear equations. The magnetic field gradients at the dipoles placed close to the actual targets should be greater than those farther away.

The results for a simple test case showed that the model could identify the correct location for the dipole. However when used on measured data with the effects of the earth's magnetic field and noise, then the results are not always clear. The best result was obtained by solving the equations for five dipoles located directly below five data points and then using the median value of the five results for each dipole as the window is moved along the data. By taking the median values the effect of the earth's magnetic field is removed and we are left with the four targets that are found with the Frahm method.

The proposed model does work for locating targets and has the advantage over the Frahm inversion in that it can be used on the uncompensated data. However, there is no reason that the settings used to obtain the best result for a set of data will always be the best setting for any given data. In fact, slight changes in the settings can yield very different results. Thus, although the model can locate targets, it is not a very practical method for real world data.

Frahm, C.P. "Inversion of the magnetic field gradient equations for a magnetic dipole field", Naval Coastal Systems Laboratory Rep. NCSL 135-72, Panama City, Florida, 1972.

# OPTIMAL ORDER QUANTITY WITH VOLUME DISCOUNTS BEFORE AND AFTER PRICE INCREASE

#### Super Group, Supply Chain Management

#### **Industry Representatives**

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#### Moderator

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#### Description

In the buying and selling business there are ordering costs, holding costs and an out-of-stock penalty. A quantity discount is given if the order is sufficiently large. At the next ordering the purchase price increases by a certain percentage. The Study Group was asked to determine how much to order now at the lower price and in the proceeding cycles at increased prices to maximize net profit.

#### **Executive summary**

The Study Group observed that some economic order quantity (EOQ) models with quantity discounts and optimal order quantity models with stochastic demand and leadtime without quantity discounts have been proposed and discussed in the literature. Work has also started on the EOQ model with price increases. The Study Group proposed that a possible approach would be to investigate how the various models could be adapted and/or combined to model the problem at hand. A model that addresses the case of prise changes has been formulated and tried on some data. Work is continuing to incorporate volume discounts in order to address the other aspects of the problem.

### SCHEDULING OF MATERIAL THROUGH A STEEL PLANT

#### **Columbus Stainless Steel**

#### **Industry Representatives**

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#### **Moderator**

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#### **Description**

One of the main problems in any steel plant is the scheduling of material through the plant. The Study Group was asked to develop an optimization model with the objective to optimize the scheduling of the cold rolling mills. The input is the processing times for the different steel types at the different mills.

#### **Executive summary**

The scheduling problem has two buffers. A buffer is a stock pile of steel consisting of various types, widths and lengths before the steel is processed through the different mills. The Study Group produced a simulation model of the first buffer which gave an indication of alloy types and their start and end gauges. The optimization of the sheduling of the cold rolling mills was then considered. The constraints were the processing time per alloy type and the capacities of the mills for handling input and output gauges. Work on the problem is continuing.

The simulation and optimization together will enable the scheduler to quickly evaluate stocks in the buffers and their optimal scheduling.

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